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THE MATHEMATICS TEACHER

Volume XLII Number 8

Edited by William David Reeve

The Certification of Teachers of Mathematics[1]

By W. I. LAYTON
Dean of Instruction, State Teachers College, Frostburg, Maryland

THIS paper is based upon a part of *An Analysis of Certification Requirements for Teachers of Mathematics*, Bureau of Publications, George Peabody College, Nashville, Tennessee, 1949. We will deal with the requirements for the certification of mathematics teachers for Grades I through XII in the forty-eight states and the District of Columbia. We will also consider some means whereby the certification of mathematics teachers may be made more effective.

The rules and regulations on certification issued by the office in charge of certification in each of the forty-eight states and the District of Columbia were first carefully analyzed. A compilation of elementary school certification practices including academic and professional requirements was made. The following were also included wherever they were specified in the state bulletins: recommendation from college, a health certificate from a physician, requirement of the National Teacher Examinations, and minimum grades in the subjects to be taught and in professional training.

In order to compare mathematics with other content courses included in the preparation of elementary teachers, an analysis was made of the minimum requirements in semester hours for the lowest and highest initial elementary school teachers' certificates in effect as of January 1, 1947. The following subjects were included: mathematics content, methods in mathematics, English content, geography content, art content, professional courses excluding practice teaching, and practice teaching. The data also contained the minimum college years required for these certificates. Nation-wide means were computed for all the above-named topics for both the lowest and the highest initial elementary certificates. The term "initial" refers to certificates for applicants without teaching experience. The lowest and highest classification is used only for regular teachers' certificates based on credits. It does not pertain to emergency or temporary certificates.

An analysis for the lowest and highest initial secondary school teachers' certificates similar to that previously described for elementary certificates was also made. However, the subjects used for comparison on the secondary level were: mathematics, English, history, and biology content in addition to the professional training and the minimum college years required. Here again, nation-wide means were computed. The same type of analysis

[1] Read at the twenty-seventh annual meeting of The National Council of Teachers of Mathematics at Baltimore, Maryland, April 1 and 2, 1949.

was applied to certificates which are issued only for use in junior high schools.

Next a questionnaire was sent to the certifying officer in each state and the District of Columbia. It included (1) All Levels of Certification, (2) The Elementary School, and (3) The Secondary School Excluding the Junior College. The questionnaire was primarily concerned with determining the opinions of the certification officers on the number of hours in content mathematics which should be included and the professional training necessary for teachers of mathematics on the levels just named. Other factors such as minimum grade, recommendation from college, and a health certificate, all of which enter into the certification of teachers of mathematics, were included. Results of the questionnaire were analyzed and the points on which a majority of the certification officers agreed were then submitted to mathematics specialists throughout the nation who are interested in the training of teachers of mathematics.

An analysis of the present situation in the elementary school reveals that:

The requirements in content mathematics for the lowest and highest initial elementary certificates are lower by far than the requirements in English, geography, and art. Some nation-wide means for the lowest certificate, expressed in semester hours, are: mathematics content .52; methods in mathematics .16; English content 3.92; geography content .87; art content 1.13; professional courses excluding practice teaching 9.57; practice teaching and observation 2.60; total of all professional courses 13.01 semester hours. The means for the highest certificates differ very little from those just given for the lowest elementary certificates and so are not listed in this brief paper.

Very few of the states require any training in either mathematics content or methods, only ten specifying any training in mathematics content for the lowest initial elementary certificate and no state requiring more than five semester hours in mathematics content for this certificate. For the highest initial elementary certificate only eleven states call for training in mathematics content with five hours again being the greatest number required. Only five states stipulate training in methods in mathematics.

Many more states are concerned with professional courses in the training of elementary teachers than with mathematics content courses.

The variation in mathematics content requirements is quite wide, from zero to five semester hours. Specific courses named in the bulletins are general mathematics, teachers' arithmetic, arithmetic, and practical arithmetic.

There is a fairly high degree of uniformity in the total professional requirements of the states for elementary teachers of mathematics but there is little agreement concerning the specific courses to be included in these totals.

There is little difference in the mathematics content requirements between the lowest and highest initial elementary certificates.

An analysis of the present situation in the secondary school discloses the following facts:

Almost 30% of the states require no training in content mathematics for secondary school teachers of mathematics on initial secondary school certificates.

The requirement in mathematics content courses for mathematics teachers ranges from zero to twenty-four semester hours with a mean of approximately ten semester hours.

Only five of the states specify any particular mathematics courses and among these states there is little consistency as to course titles. Algebra is prescribed by two states, geometry by two, trigonometry by five states, mathematics of finance by one, college algebra by three, college algebra or freshman mathematics by one, solid geometry or college geometry by one, analytic geometry by two states, calculus by two, and college geometry by one state.

Six states permit deductions from the required mathematics courses on the basis of credits received for mathematics in high school.

Only one state has a requirement concerning training in related fields for mathematics teachers.

The total number of hours prescribed in mathematics for teachers of the subject is somewhat lower than that in English content for English teachers.

All the states require professional training but the only specific course prescribed by 50% or more is practice teaching.

The professional requirement for mathematics teachers is nearly twice the mathematics requirement for the same teachers.

Only two states indicate that the secondary school mathematics teacher is required to take methods in that subject.

Very few states require a minimum or average grade, recommendation from college, or a certificate from a physician.

A majority of the states require a minimum of four years of college for secondary certificates.

Only one state uses the National Teacher Examinations as a factor in certification.

Five states issue certificates to be used specifically in the junior high school and their professional requirements are almost three times that of the mathematics content requirements.

Let us now turn to some recommendations. These recommendations are based on the approval of a majority of the two groups receiving questionnaires, namely, the certification officers and the mathematics specialists. The following recommendations are made with respect to the particular area designated:

All Levels of Certification

The requirement of either a definite minimum grade in each course or a minimum average grade in all mathematics courses submitted for certification

This practice would eliminate the possibility of a candidate being certified to teach mathematics when he had a "D" average in his mathematics courses, for instance.

A recommendation from the college concerning the teaching competency of the applicant

A certificate from a physician

Not requiring the National Teacher Examinations of the American Council on Education

No certification of teachers by the Federal Government

Elementary School

It is understood that elementary teachers, as a general rule, give instruction not only in mathematics but also in the various other academic subjects. Concerning the grade span, it should be borne in mind that there are still many school systems which include the first eight grades in the elementary school. In view of the preceding statements, the following recommendations are offered:

A bachelor's degree

A minimum of six semester hours in content mathematics consisting of a college course in arithmetic and one in general mathematics

The college course in arithmetic should be such as to insure competence over the whole range of subject matter which is usually included in elementary school arithmetic. The course in general mathematics should be non-technical in nature dealing with topics in algebra, geometry, trigonometry, business problems, and statistics. As an alternate to these courses the candidate for the elementary certificate should perhaps be permitted to submit six semester hours in college algebra and trigonometry.

Professional courses including practice teaching, educational psychology, observation in teaching, principles of education, general psychology, general methods of teaching, and the curriculum

SECONDARY SCHOOL (EXCLUDING THE JUNIOR COLLEGE)

Requirement of a bachelor's degree but not a master's degree

Eighteen semester hours in content mathematics for the minimum including college algebra, trigonometry, plane analytic geometry, solid geometry, and calculus.

Eighteen semester hours in professional courses for the minimum including practice teaching, educational psychology, general psychology, the teaching of senior or junior high school mathematics, observation of teaching, and tests and measurements

Training in related fields especially physics, chemistry, economics, and astronomy

"Training in related fields" should be interpreted in its broadest sense. Surveying, slide rule, engineering drawing, and trade and business applications are other areas which are important.

Let us now consider a brief summary of the material just described, keeping the requirements in mathematics foremost in our minds. The nation-wide mean of content mathematics required of those who will teach in the elementary school is .52 of a semester hour. This is .78 of one quarter hour. This means that a vast majority of our elementary teachers of mathematics have had, as far as requirements are concerned, no mathematics except what they themselves studied in the elementary school. Those who teach English in the elementary school are not permitted to do so on the sole basis of their English training received in the elementary school. Those who teach social studies in the elementary school are not allowed to do so on the sole basis of their social studies training received in the elementary school. No! Far from it! Why is this? Is mathematics such an easy subject? The consensus of opinion seems to be "no." Mathematics teachers need to have far better training than .52 of a semester hour will give them. Why does this situation exist? Partially because people like you and me who are interested in mathematics let it exist. It is up to us to exert ourselves to see that competent mathematics instruction is given in elementary school mathematics.

This report recommends as a minimum six semester hours or nine quarter hours in content mathematics for teachers in this area.

In the secondary school also the situation is chaotic. As already pointed out, almost 30% of the states require no training in content mathematics for secondary school teachers of mathematics, on initial secondary school certificates. The nation-wide mean is approximately ten semester hours. This paper recommends as a minimum 18 semester hours or 27 quarter hours in content mathematics for these teachers.

What can we do in the face of the low requirements described? We can apply a little concentrated effort. Organizations like the National Council of Teachers of Mathematics carry much more weight with the certifying agencies than we might think on the surface. I would like to see The National Council of Teachers of Mathematics appoint a committee to work with the Mathematical Association of America and with the Mathematics Sections of the State Education Associations in an effort to improve the certification rulings which affect those who teach mathematics on the different levels.

A Functional Program for Secondary Mathematics*

By William A. Gager
University of Florida, Gainesville, Fla.

In your motoring have you ever had the experience of coming to a fork in the pavement, and, not being able to make up your mind which road to follow, selected the one that seemed the best to you, only to find later that it came to a dead end down around the curve? An occurence similar to this has been experienced by many of our ninth grade mathematics students over a period of many years.

Several years ago we branched away from our traditional secondary mathematics curriculum on the ninth grade level, with a new course. Among other things it was hoped that this course would present a body of materials that would reduce the number of failures in traditional ninth grade algebra; would provide materials within the interest and understanding of the poorer students; and would provide a course of special value to those students who would not take mathematics after the ninth grade, regardless of whether or not they remained in school. Much effort has been expended to make this course effective, even to the extent of giving it attractive names such as applied mathematics, consumer mathematics, economic mathematics, essential mathematics, everyday mathematics, general mathematics, vital mathematics, home mathematics, living mathematics, useful mathematics, and so on, until the count has about exceeded Heinz's "57" varieties! No matter though how easy and inviting was the entrance to this course, or how appealing and attractive was its name, general mathematics in the secondary mathematics curriculum has always been a dead-end road at the end of the ninth year, and a rather poor one at that. After more than a quarter century of effort we still must admit the fact that as yet we do not have a good general mathematics program for students of any type—be they poor, average, or good students.

You might wonder why I bring "good students" into this discussion when, as I have previously stated, the original idea was to prepare a good general mathematics course for the poorer ones. The point is that in late years they are found in appreciable numbers in every general mathematics class, and therefore are an important part of the total problem to be considered. They get in these general mathematics classes either by accident or by choice.

Competent guidance at the beginning of the ninth grade has been so rare that the assurance of a student's being registered for the proper mathematics course verged on pure accident. Even the mathematics teachers have failed them at this point. Too many of our mathematics teachers glorify algebra and geometry and look down on general mathematics; they guide to the extent of trying to skim off the cream of the brains for their algebra classes; they guide to the extent of pushing the larger numbers into general mathematics, a very inferior course in their minds. This is not adequate student guidance. Moreover, this teacher attitude has contributed greatly to the present state of general mathematics.

On the other hand let us not overlook the fact that many of our better mathematics students get into general mathematics by choice. This is a commendable choice too and is in complete accord with one of our more recent basic reasons for a good ninth grade general mathematics course; that is, "to provide a mathematic-

* Read at the twenty-seventh annual meeting of The National Council of Teachers of Mathematics at Baltimore, Maryland, April 1 and 2, 1949.

ally sound ninth grade course so organized that it could be required of all high school students who did not specifically choose traditional algebra."[6] Our brighter mathematics students certainly could benefit as much from good work in a well developed general mathematics program as from the formal algebraic and geometric approaches to mathematics. If by their own choice, or as a result of proper counselling, they prefer this more specialized and functional type of mathematics, such courses should be provided for them.

When one considers the many capable men and women who have labored to make ninth grade general mathematics what it should be, he finds it hard to understand why so little progress in the right direction has been made. I can think of at least three hindrances.

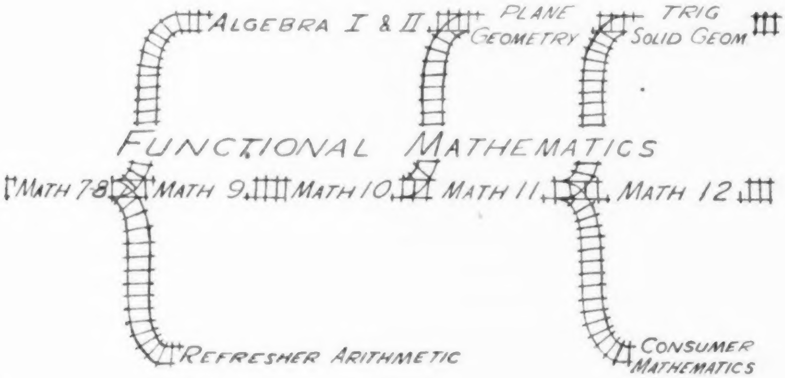
1. Poor teaching, inadequate guidance, and wrong attitude on the part of many teachers and administrators have had a very demoralizing influence.
2. We have so oversocialized this ninth grade general mathematics that the basic mathematical concepts are either not brought in at all or, if inferred, are so covered up with a wilderness of words that they can't be detected.
3. Realizing that it would be our last "mathematical shot" at these ninth graders we "loaded" this general mathematics course in such a manner that the task of teaching the subject tended to "floor" our teachers and at the same time too many of our "shots" missed the students.

Of course the sixty-four dollar question is, "what can be done to better the situation?" One effort to obtain a better solution was instituted in the summer of 1948 when the University of Florida and the Florida State Department of Education brought to the University under the direction of this author a group of twenty-five carefully selected secondary mathematics teachers. One of the responsibilities of this group was to recommend revision needs in regard to general mathematics, and to prepare materials in line with the revision recommendations.

After detailed and careful weighing of the current general mathematics content, a study of the mathematical literature bearing on general mathematics, and a consideration of the judgment of teachers and administrators who had had experience with general mathematics on the ninth grade level, the Revision Committee came to a unanimity of thinking on these points:

a. that the individual learns only that which has meaning for him, that which satisfies a need for him, and that which he can be made to feel will prepare him to meet his future needs.
b. that to contribute successfully to this learning process, mathematics must provide experiences which are socially significant, personally satisfying, functionally worthwhile, and mathematically sound.
c. that, if secondary mathematics is ever going to meet successfully the demands of modern life for most people, it must be made more functional in grades 9 through 12.
d. that the present traditional mathematics with a general mathematics for ninth grade only is out of joint with the times.
e. that the main-line track for mathematics through high school should be reserved for a sequence of mathematically sound functional courses to become known as Mathematics 9, Mathematics 10, Mathematics 11, and Mathematics 12.

The diagram which follows shows the proposed functional courses occupying the main-line track, while the traditional courses follow in sequence on the side line. Here, as in the past, the traditional courses are recognized as key courses for certain



students and for certain types of future work. In view of the fact, however, that few take the traditional courses, in contrast to the many available for the functional courses, the arrangement appears worthy of careful consideration.

If it should be that a few ninth grade students are so totally deficient in mathematics that the functional course cannot benefit them in any way, then the school might consider offering this group refresher arithmetic in the ninth grade and consumer mathematics in the twelfth grade. In general I think the arrangement should be avoided if at all possible. If it is used then the students assigned to these courses should be very few in number and very carefully chosen.

The development of a sequence of functional courses for secondary mathematics, as suggested by the diagram and as was worked out in considerable detail by the Florida Revision Committee, is not a new idea. The need for a functional approach has been discussed by leaders such as Betz,[1] Brueckner,[3] Douglass,[4] Price,[9] Reeve,[11] Schult,[12] and many others, over a period of several years, but not enough has been done about it.

Brueckner writes,

> It seems clear that in the future there must be a general reconstruction of the curriculum in mathematics at the high school level if the subject is to be made functional and if mathematics is to make the contribution it should make to the management of the affairs of a democratic society.[3]

Along "this line Betz says,

> If we really desire to have a functional two-track program at the secondary level our curriculum makers and administration must be prepared to set aside two years for a course in general mathematics intended for the other 85%. Anything less than that will simply continue or repeat the futile contortions of the past.[1]

As recent as February 1949 Reeve, in discussing "General Mathematics For Grades 9 through 12," writes,

> I know, as a matter of fact, that, for many pupils and their teachers, the sequence of algebra, geometry, trigonometry, and so on will continue to be that chosen in an appreciable number of schools and often for reasons that may seem valid enough, at least to a great many teachers. If we can agree on that, we may then view honestly and fairly the question as to whether, for large numbers of our pupils, a reorganized course in general mathematics may not be more profitable for all concerned.[11]

These quotations, and the many similar ones that could be added, are sufficient to indicate that the Florida Revision Committee, in deciding to develop a sequence of functional mathematics courses, has taken hold of a real vital problem.

Because the sources of mathematical powers which enable one to develop more effectively as a citizen and which furnish him with a basis for acceptable contributions to society are anchored on basic mathematical concepts, principles, procedures, and skills; therefore, the Revision Committee felt that the most urgent problem facing it was to prepare for each grade on the secondary level an itemized list of mathematical concepts and principles, a grade placement chart, and a list of suggestive content.

The task of isolating the basic mathematical concepts which the Committee felt should be included in functional courses proved to be an extremely fascinating one but also a most difficult one. One can appreciate the difficulty better by trying to react concisely and precisely to such ideas as one-to-one correspondence, numbers, exact numbers, approximate numbers, signed numbers, zero, infinity, sum, difference, product, quotient, series, percentage, limits, lines, planes, surfaces, circles, diameters, triangles, quadrilaterals, cylinders, spheres, lengths, areas, volumes, angles, time, ratio, formula, equations, proportions, variations, probability, and so on. This list, which now contains approximately 160 items, counting all the sub-heads, had to come first in our thinking for two good reasons: (1) these concepts were to serve as the foundation for the proposed functional courses, (2) these concepts provided a means of evaluating content material in current ninth grade textbooks. By means of these concepts as a check-list some of the textbook materials were found to be so lacking in mathematical ideas and development that they were definitely omitted from materials to be considered for the functional courses.

It was recognized, of course, that mathematics all chopped up into many single ideas was not in its best form to function. The Committee therefore proceeded to give thought to isolating the basic and working mathematical principles which would be needed to make the courses function.

A sample of the list of principles, sufficient to give a conception of what the total list contains, follows:

1. The position of the digits in a number determines the value of the number.
2. The numerator and denominator of a fraction may be multiplied by the same number (except zero) without changing the value of the fraction.
3. Division by zero is impossible.
4. The elements in addition and multiplication may be combined in any order without changing the result.
5. The accuracy of a product of a quotient is no greater than the accuracy of its least accurate factor.
6. When measurements involving different units of measure are to be added the sum should claim no greater precision than the least precise of the measurement.
7. Both members of an equation must be acted upon by the same mathmatical operation, otherwise the balance will not be maintained and the equation will destroyed.

It is a widely accepted belief that the secondary mathematics program would be greatly improved if significant materials could be placed in those grades where such materials would have most meaning to the students. In line with this thinking the Committee, in its organization of materials, scheduled the concepts from the seventh through the twelfth grades according to their importance and according to the degree of development desired in each grade. Also, the Committee thought it wise to move some of the materials now presented in the ninth grade, to the tenth, the eleventh, and even the twelfth grade. A strenuous effort was made to see that each concept was developed as far as justified for functional courses on the secondary level, and, to see that certain phases of each concept were developed in the grade level most compatible with the learning readiness of the learner. This effort to determine the degree of development of each concept on each grade level very naturally led to the building of the Grade Placement Chart for grades seven through twelve, and, this chart in turn served as the basis for the selection of actual content materials for each grade on the secondary level.

In regard to content materials, the Revision Committee found time to provide in tentative form, ample materials to make it possible for interested teachers to

try out the suggested content on the ninth and tenth grade levels. Materials for grades eleven and twelve also were given much attention but for the time being were prepared more for purposes of criticism than for immediate use. All these materials will be weighed anew and greatly refined as the study continues through 1949.

These functional courses have been planned in such a way that a high school student *must* complete Mathematics 9 and Mathematics 10 in order to reach that degree of competence sufficient for present day living. This means that Mathematics 9 and Mathematics 10 should be required leaving Mathematics 11 and Mathematics 12 as electives for those who want to become more proficient in mathematical thinking and applications. In Florida, the State Department of Education has expressed a willingness to require Mathematics 9 and 10 if and when the results obtained from experience with the materials, conclusively show that such action should be taken.

As so often happens, striving for improvements in one direction often led to problems in another direction. In working on these functional courses the problem of the adequate teaching of them was brought to a very keen focus. It is an acknowledged fact that it requires superior ability and energy to teach successfully functional courses in mathematics. The very nature of these courses demand good teachers. The functional program cannot succeed without them. Our Committee, therefore, feels that improvement of the curriculum and the materials is not enough. An improved teacher training program for secondary mathematics teachers is also essential.

It is the carefully considered judgment of the Florida Revision Committee that the functional courses, Mathematics 9, 10, 11, and 12, here discussed can, if supported by good teaching, prove to be one of the most valuable contributions to our secondary mathematics curriculum made in recent years.

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A Merry Christmas and a Happy New Year!

The Relations of Secondary Mathematics to Engineering Education*

By S. S. Steinberg
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In discussing the topic assigned I have tried to be as informal in my presentation as possible. The topic is not new, but has been debated, often heatedly, many times. On the one hand, we have the high school which tries to prepare students for many callings, and on the other the engineering college which has the specific job of training young men in a minimum of time to become qualified professional engineers. You all fully realize that a thorough knowledge of mathematics, at least through calculus, is fundamental to all engineering education, and that the mathematical knowledge the student brings with him from high school is the foundation upon which our usual engineering curriculum of four years is built. If the freshman brings the prerequisite basic mathematical knowledge with him, we can complete our structure in the allotted time. If not, then as in all engineering work, failure to complete on time brings with it a heavy penalty. For many years, the mortality of engineering college students throughout the United States has stood approximately at 60%; in other words for every 100 students that enter a college of engineering in the fall about 40 graduate at the end of four academic years. The reasons for this high mortality are many, but in most cases it is due to lack of ability in mathematics, in spite of the fact that most students enter the engineering college with the required mathematics from high school.

In the larger secondary schools with their larger staff of teachers, greater attention can be given to those groups of students who plan to enter a particular field or profession, such as engineering. In the smaller schools, this would be out of the question and therefore the student in these schools who plans to enter a technical or scientific field may not have the portunity to prepare himself fully in the fundamental subjects, such as mathematics, to enable him to make the progress he should when he comes to college. Insofar as the profession of engineering is concerned, this disparity of educational opportunity may, in some cases, militate against the country boy and favor one who comes from the city with its broader educational facilities. If this is so, it is quite regrettable since our experience shows that the young man coming from the farm and the smaller communities often displays unusual mechanical ability and ingenuity. This last word "ingenuity" is, after all, the root of the word "engineering."

Many years of contact with engineering education brings one to the conclusion that almost from the day of registration of the freshman in an engineering college it is possible to predict, other things being equal, whether a particular young man will be graduated with a degree. The student who has done well in mathematics in high school, the student who likes, or at least does not dislike mathematics, the student who is attracted by the physical sciences, such as chemistry and physics, is the student who will generally graduate in engineering. Conversely, the student who is poor in mathematics in high school or who has no special liking for the subject, but is attracted only by the glamour of the profession, should not enroll in an engineering college. In most state institutions, like our own, where pre-selection for mathematical ability is not made, those students who have the requisite

* Read at the twenty-seventh annual meeting of The National Council of Teachers of Mathematics at Baltimore, Maryland, April 1 and 2, 1949.

entrance credits in mathematics are scheduled during their first semester in engineering in College Algebra, 3 credits, and in Plane Trigonometry, 2 credits. Solid Geometry, $\frac{1}{2}$ credit, is required by us for entrance, but if the student's high school does not offer the last course or he has failed to take it, we require him to schedule it during the first semester.

Every engineering student in our institution is required to take during freshman week, a qualifying test in mathematics to determine whether he is adequately prepared for the freshman courses in College Algebra and in Plane Trigonometry. A student failing this test is required to take Introductory Algebra, without credit, and is not eligible to take Plane Trigonometry concurrently. If the student is deficient in Solid Geometry for entrance, he takes it concurrently with the Introductory Algebra, if he has failed the qualifying test. If he has passed the test, he may make up the Solid Geometry in the succeeding summer school. Students exceptionally capable in mathematics may carry the Solid Geometry deficiency concurrently with the College Algebra and Plane Trigonometry in the first semester of the freshman year.

Much experience with freshman engineering students leads one to the belief that mathematicians, like poets, are born and not made; at least, that people are born with mathematical ability and that it is not a quality which can be acquired with the passage of time. It appears that the old adage which counsels "try, try, again" does not seem to apply to mathematical ability and rarely produces results if the native ability is not there. This is one field of endeavor in which persistency is no virtue. The choice of engineering as a career by students not qualified, results in the crowding of freshman engineering classes with students who do not long remain with us. The result is a loss in many ways to the student and to the engineering college. Unqualified students are compelled to leave, often with a feeling of shame and inferiority, because of lack of accomplishment which is not wholly their fault. There is a loss to the capable student whose pace has been slackened, and to the engineering college in expending funds on temporarily expanded classes and staff.

There have been many discussions regarding the varying requirements for admission to engineering colleges as it concerns Solid Geometry and Plane Trigonometry. Some require the former for entrance, others the latter, and some will accept either. Recently we made a survey of available catalogues to get some statistical data on this subject. We found that of State universities, 27 require Solid Geometry for entrance to their engineering college, while 9 do not; of private institutions, 7 require it, while one does not. The area from which we draw students includes Baltimore and Washington, D. C., and all the high schools in these two cities offer Solid Geometry. Out of 150 county high schools in Maryland only 32 offer Solid Geometry.

Of all high school subjects, mathematics probably has the least glamour or appeal to most students. It is undoubtedly less romantic to them than history, geography, or Shakespeare. In mixed classes, it usually has little appeal to the girls, and if the boys find it difficult too, because it requires digging in and solving problems, it loses its charm in any popularity contest. Then, too, is the average high school student when taking such courses as Algebra, Solid Geometry, Plane Trigonometry, sufficiently mature to really grasp mentally the basic concepts of these courses? Is he able, at that stage in his development, to concentrate sufficiently so as to anchor the fundamental principles to experiences that he has lived through? Or is he trying in many cases merely to memorize, to parrot, and to cram, just sufficiently to get through the course to which he can then say "Amen"?

And what about the other half of the team, the high school mathematics teacher? Does he or she know thoroughly

and love the subject being taught? Is the subject matter being made attractive and as interesting as it might be? Does the teacher continuously point out to the student the value of mathematics not only in specialized study, such as engineering and technical fields, but also in every day life in an engineering age? This being the National Council of Teachers of Mathematics, this group is undoubtedly in a better position to answer these questions than I am.

In conclusion, it is believed that the relation of secondary mathematics to engineering education would be better understood and would result in greater benefit to both groups concerned, if closer relations existed between the teachers of high school mathematics and the faculties of engineering colleges. As in international relations so in inter-faculty relations, frequent meetings and discussion of common problems make for better understanding and mutual helpfulness. A closer intermingling between high school and college teachers in each area would result in greater cooperation and better preparation of high school students for engineering and technical education. The result would be less economic waste as well as discouragement to both the student and the engineering college.

Amicable Numbers and the Christmas Message

By A. N. Tucker

J. Sterling Morton High School & Junior College, Cicero, Illinois

A number which is equal to the sum of its factors, excluding itself, is called a perfect number. 28, which is equal to $1+2+4+7+14$, is an example. There is another one easy to guess, but they are scarce,—as one might expect anything perfect to be. The notion of relationship between a number and its divisors appeals to a mathematician and hence the rare instance where of two numbers, either one being equal to the sum of the divisors of the other, not counting the numbers themselves among the divisors, is particularly pleasing. 220 and 284 are such a pair and for this felicitous property are known as amicable numbers. It is a nice idea, each composing and being composed by the other without losing its own identity.

We all strive for perfection and in the course of life to achieve an amicable association with our fellow creatures. In the field of human relationships, definitions are not as precise as in the realm of numbers, and the factors can not readily be discerned. Yet we may find the human counterpart of this mathematical pair in the two phrases of the Christmas message, where each idea forms a basis for the other and where together they point man toward perfection.

PEACE ON EARTH

★

GOOD WILL TO MEN

A Study in Needed Redirection in the Preparation of Teachers of Arithmetic*

By Vincent J. Glennon
School of Education, Syracuse University

A few years ago in a city in New England there appeared on the front page of the newspaper the story of an investigation of the death of a hospital patient. This story received more than the usual amount of publicity for such an incident because of the fact that the patient was not in any serious condition immediately prior to his death. To the medical authorities the cause of the death was not readily apparent, and an investigation was carried out. Among other things the investigation revealed that the patient was administered a drug shortly before his death. Further investigation of this particular act brought to light the fact that a nurse had been requested by the medical doctor in charge to prepare a solution using *one-eighth* of a grain of a particular drug. Pursuing the investigation still further, the nurse was asked to *tell* and *show* exactly what she had done in preparing the solution. In describing her behavior the nurse said that she did not know what *one-eighth* was, but she thought that since *four* and *four* made *eight*, *one-fourth* and *one-fourth* should make *one-eighth*. And she prepared the solution accordingly!

The tragic aspect of this incident is equaled only by the high degree of mathematical illiteracy represented in the incident.

The layman might well ask "How could such an error occur?" "How could it happen that the child having become a nurse make such a grave error?" "How could it happen that she did not have a correct understanding of the fraction 'one-eighth'? " The answer to this latter question is not simple and single, but rather subtle and complex. The whole anwer might include such factors as the ability of the learner to acquire the understanding, the quality of the social climate in which the learning occurred, the kinds or types of learning activities in which the students participated, the attitude of the teacher toward the methods of teaching, that she was using, the understanding of the teacher of the relative effectiveness of different methods of teaching, the appreciation on the part of the teacher of the type of situation in which learning occurs best, and the degree to which the teacher, herself, had acquired those mathematical understandings and meanings which are basic to the computational processes commonly taught in grades one to eight.

This paper is concerned with reporting a study of the last of these factors: the degree to which student teachers and teachers-in-service have acquired the basic mathematical understandings and meanings.

The paucity of *precise* studies in this area of teacher education that have been carried out in the last thirty years is disconcerting indeed. The writer found only one *study*[1] that was related to the problem, and it was written over ten years ago. All other published writings in the area of *teacher education* in *arithmetic* seem to be concerned with our telling each other what we *should be doing* in the redirection of the program.

* Read at the twenty-seventh annual meeting of The National Council of Teachers of Mathematics at Baltimore, Maryland, April 1 and 2, 1949.

[1] Taylor, E. H. "Mathematics for a Four-Year Course for Teachers in the Elementary School," *School Science and Mathematics*, May, 1938.

The Present Study: Its Purpose, Method, Findings and Implications[2]

Purpose

It was the purpose of the study to determine the extent of growth and mastery of certain basic mathematical understandings possessed by three groups of persons: Teachers College freshmen, Teachers College seniors, and Teachers-in-service. The total group included 476 persons, of which 144 were freshmen, 172 were seniors, and 160 were teachers-in-service. The college students were enrolled in three Teachers Colleges. All were preparing to teach on the elementary school level.

The teachers-in-service were teaching in grade kindergarten through grade eight. The numbers of years in service ranged from one to thirty-four years.

Method

The general type of investigative method used in the study was the normative method; that is, a method through the use of which we obtain an index of prevailing conditions within the groups of persons being studied.

The specific method used in gathering the data was the administering of a *Test of Basic Mathematical Understanding.* The first job, then, was the selection of the instrument to be used in gathering the data. For this the researcher turned to a collection of arithmetic texts that included all tests published since 1915. Each test was carefully examined for the purpose of determining the number and quality of items designed for the purpose of measuring *basic mathematical understandings.* The examination of the tests revealed no tests that would meet the need of the study.

Reflecting this need for newer instruments Butler[3] said, " . . . it is certain that in the domain of published tests the specific testing for mastery of mathematical concepts (understandings) has received scant attention."

Brownell[4] had this to say on the same point: "Exceedingly little has been done either informally or systematically to find practicable and valid procedures for evaluating the outcomes (in Mathematical Understandings)."

It was necessary then to construct an instrument that could be used in the study for gathering the data. Stated negatively, the new instrument must be of such a kind as to eliminate entirely or at least minimize the effect of rote computational facility as a determiner of success. A test of mathematical understandings that involved computation habits, which may have been learned largely through repetition, would be quite invalid, since it would be difficult to determine the degree to which the testee's responses were the result of *understanding* or the result of *rote memory.*

This statement does not carry the implication that tests of computational facility such as those examined do not require understanding for success, or that pupils who do not understand will be as successful as those pupils who do understand. On the contrary, it is quite probable that the pupil with the greatest degree of understanding will be the most successful on the test of computational facility. However, it is true that the test which eliminates or minimizes computational facility will also eliminate or minimize one factor contributing to the nonvalidity of the instrument as a measure of understanding of number relationships. For this reason the instrument was con-

[2] Data included in this paper taken from a larger study: Glennon, Vincent J. "A Study of the Growth and Mastery of Certain Basic Mathematical Understandings on Seven Educational Levels." Unpublished doctor's dissertation. Cambridge: Graduate School of Education, Harvard University, 1948.

[3] Butler, C. H. "Mastery of Certain Mathematical Concepts by Pupils at the Junior High School Level," The Mathematics Teacher, March, 1932, pp. 117–172.

[4] Brownell, W. A. Sixteenth Yearbook National Council of Teachers of Mathematics. *Arithmetic in General Education*, 1940, pp. 247.

structed in such manner as to eliminate all direct computation.

Another characteristic of the new instrument is objectivity. To attain this characteristic the test items were built in the form of multiple choice items. This type of item was chosen on the basis of such statements as that of Hawkes, Lindquist and Mann:[5] "The multiple choice type is perhaps the most valuable and the most generally applicable of all types of test exercises." Included among the choices in the test items were typical pupil responses. These were obtained by the administration and analysis of test items of the completion type—that is questions in which the testee was not given any aid or clues in recalling the correct answer.

Several commonly used validation procedures were considered for their applicability to the problem of constructing this particular instrument. There is time here to but list the procedures: the first validation procedure was concerned with analytical or curricular validity; the second procedure considered was that of determining the statistical validity of the instrument by comparing scores made by testees on the new instrument and a criterion instrument; a third technique commonly used and considered for this study is that which is based on the increase of scores with increasing grade levels. Still another technique commonly used and considered here is that type of validation procedure which makes use of the combined judgments of relatively equally well informed experts in the field covered by the test.

No one of these techniques for determining the validity of an instrument is useful by itself. In the last analysis the best method for determining the validity of any instrument is an observation of the behavior of the testee—keeping in mind the question: Does the test distinguish between the behavior of the person who understands (or has a given ability, or has a given attitude, etc.) and the behavior of the person who does *not* understand?

With this question in mind the researcher worked with several groups of children—two children per group—on the seventh grade level over a period of two months. It was felt that working with two children would make for better rapport between the children and the researcher, since a single child might find it difficult to adjust readily to the new situation.

"Loaded" questions were discovered by the child's ability to select the correct answer without being able to describe or verbalize the understanding involved. Such questions were either revised or eliminated. This procedure was repeated with many groups of children until they identified no new "loaded" questions.

During the procedure the tester also noted evidences of reading difficulties stemming from the wording of items. Whenever the item seemed to cause such difficulty, the children were asked to tell what they thought the item meant, and were asked to state the meaning in their own words. This wording was noted and the items changed to conform with wording that was more meaningful for children. Difficulties in sentence structure, vocabulary burden, and ambiguities were also noted and the items modified or eliminated.

Through this procedure the original test of 136 items was reduced to 90 items. A test of 90 items of this ilk is somewhat too long for use in a single classroom sitting, and the number of items was reduced to 80. The items covered five areas of basic mathematical understandings: I. The decimal system of notation (15 items), II. Basic understandings of integers and processes (15 items), III. Basic understandings of fractions and processes (15 items) , IV. Basic understandings of

[5] Hawkes, H. E., Lindquist, E. F., and Mann, C. R. *Construction and Use of Achievement Examinations.* Boston: Houghton-Mifflin Co., 1936, p. 138.

decimals and processes (20 items) and V. Basic understandings of the rationals of computation (15 items).

A preliminary 'run' on the seventh grade level provided insights which aided in arranging the items in order of difficulty within each area; and also provided an estimate of the amount of time needed by seventh graders to work the test. Since the test was of a diagnostic variety, rather than achievement variety, the timing was not needed for the purpose of setting a time limit, but only as a guide for school administration purposes.

The test was administered to the Teachers College freshmen at the beginning of the school year, and to the seniors at the end of the school year. Tests were administered to the teachers-in-service at the end of the year.

Some of the Findings of the Study

The data collected through the administration of the *Test of Basic Mathematical Understandings* was collated by using several common techniques. Time does not permit of a detailed reading of the more specific findings, but some of the more general findings can be considered here.

Hypothesis I

One of the hypotheses investigated was stated this way: There is no significant difference in achievement of basic mathematical understandings between a teachers college freshman and a teachers college senior.

Findings I

The mean score of the teachers college freshman was 35.451 and the mean score of the teachers college senior was 34.186—a difference of 1.255 points in favor of the freshman. The significance ratio of .826 gives a confidence level of 41%. This confidence level supports the hypothesis that there is no significant difference in achievement of basic mathematical understandings between the two groups studied.

The findings seem to present evidence (for the groups studied) that the persons preparing to teach arithmetic in the elementary grades did not grow in achievement of basic mathematical understandings during the four years in the program of teacher education.

Hypothesis II

A second hypothesis was stated this way: There is no significant difference in achievement of basic mathematical understandings between a teachers college senior who has taken a course in the Psychology and Teaching of Arithmetic, and a teachers college senior who has *not* taken a course in the Psychology and Teaching of Arithmetic.

Findings II

Before testing this hypothesis the two groups were compared to determine whether or not there was a significant difference between the two groups in the number of mathematics courses taken in college and high school. In this sub-hypothesis the significance ratio was .137, placing the confidence level at approximately the 90% level. This level supports the sub-hypothesis that there is no significant difference between the two groups in the number of mathematics courses taken previously in college and high school.

The mean score of teachers college seniors who had a course in the Psychology and Teaching of Arithmetic was 33.984. The mean score of teachers college seniors who had *no* course in the Psychology and Teaching of Arithmetic was 34.886. A significance ratio of .379 places the confidence level at approximately 70%. This supports the hypothesis that there is no significant difference in the achievement of basic mathematical understandings between a teachers college senior who has taken the course in the Psychology and Teaching of Arithmetic and the teachers college senior who has *not* taken such a course.

Within the limitations of the study, the

evidence seems to point to the fact that a course in the Psychology and Teaching of Arithmetic did *not* bring about any significant growth on the part of the teachers college student in the area of basic mathematical understandings. This does not mean that the student did not benefit from the course in educational outcomes other than an understanding of arithmetic. On the contrary, the student may have profited considerably in acquiring a fund of knowledge about methods of teaching the subject. However, it is important to know *what* to teach as well as *how* to teach it!

Hypothesis III

There is no significant difference in achievement of basic mathematical understandings between a teacher-in-service who has done graduate work in the Psychology and Teaching of Arithmetic and one who has *not* done graduate work in the Psychology and Teaching of Arithmetic. The two groups represented the same statistical population in terms of the number of courses in mathematics taken previously in college and high school.

Findings III

Testing the hypothesis we find that the mean score of the teacher-in-service who has done graduate work in the Psychology and Teaching of Arithmetic is 42.765, and the mean score of the teacher-in-service who has *not* done such graduate work is 44.940. The significance ratio is .898. The confidence level is approximately 38% and the hypothesis is supported.

Within the limitations of the study this data indicates that graduate work in the Psychology and Teaching of Arithmetic did *not* contribute to growth in basic mathematical understandings.

Another question that was investigated in the study could be stated this way: What degree of relationship exists, among teachers-in-service in the elementary grades, between the numbers of years of experience the teacher has had in teaching arithmetic and her achievement of the basic mathematical understandings?

The correlation coefficient (Pearson) between the two variables is −.055. This is close to zero correlation, and indicates that, for the sample tested, there is almost no relationship between the number of years a person has taught arithmetic and her achievement of basic mathematical understandings.

From the findings we may conclude that the experience of teaching arithmetic is no guarantee that the teacher will grow in her understanding of the subject.

The Table is another commonly used technique for presenting collated data. Two tables are presented here:

TABLE I

Average Percent of Achievement of Basic Mathematical Understandings for Each Level

Total number of test items—80

Grade Level	Average Raw Score	Percent of Total (80)
Teachers College freshmen	35.451	44.31
Teachers College senior	34.186	42.73
Teachers-in-service	43.813	54.77

Several conclusions can be drawn from this table:

1. The Teachers College freshman understands about 44% of the basic mathematical understandings tested. These understandings are basic to the computational processes commonly taught in grades one through six!
2. The Teachers College senior understands about 43% of these basic mathematical understandings!
3. The Teacher-in-service understands about 55% (slightly more than half) of the understandings that are basic to the computational processes commonly taught in grades one through six!!!

A few statements that can be drawn from Table II are:

1. There were no (zero) items that were easy enough to be answered cor-

TABLE II
Number of Test Items Answered Correctly by Percent of Testees on Each Grade Level

Percent of Testees	Number of Items Answered Correctly by Grade Level		
	Fresh.	Sen.	Tchrs.
100%	0	0	3
90–99	5	6	4
75–89	7	11	13
50–74	25	24	32
Less than 50%	43	39	28
Total Number of test Items	80	80	80

rectly by all (100%) of the freshmen.
2. There were no (zero) items that were easy enough to be answered correctly by all (100%) of the seniors.
3. There were three (3) items that were answered correctly by all (100%) of the teachers-in-service.
4. About half of the items on the test (43 for freshmen, 39 for seniors) were so difficult for teachers college students that less than half of the students were able to answer them correctly!

The conclusions drawn from the data that have been presented thus far have not given us a very sanguine picture. The members of this audience might even be asking themselves—What kind of basic mathematical understandings were used in the test that could derive such a picture? Did we select only the very difficult understandings and omit the simpler understandings? It might be worth our while to look at a few of these understandings.

One of the easiest understandings as determined by the number of persons who selected the correct response is: Changing the order of addends in an addition example does not change the value of the answer.

The understanding that ranked about in the twentieth position in order from easy to difficult was: The third place to the left of the decimal point is the "hundreds" place.

The understanding that ranked about half way in order to difficulty was: Dividing the dividend and divisor by ten does not change the value of the answer (quotient).

The understanding that ranked in about sixtieth position out of eighty items in order of difficulty was: The denominator of a fraction tells us the number of equal parts into which the whole is divided.

One of the most difficult understandings in the test was: A digit in the 'units' place represents a value one-tenth as large as the same digit in the "tens" place.

IMPLICATIONS OF THE FINDINGS

Can we say that these understandings are very difficult understandings in the arithmetic of the first six grades? On the contrary, these understandings include some of the less abstract generalizations basic to the computational processes commonly taught in grades one through six.

All in all, the findings reported in this study do not present a very optimistic picture of the achievement of basic mathematical understandings on the part of the student teachers and teachers-in-service that were included in the study.

Recalling again for a moment the story of the nurse related at the beginning of the paper, is it not possible that the real reason for her lack of understanding of the fraction "one-eighth" may have been that the teachers who taught her arithmetic as a child did not themselves really *understand* arithmetic? And not understanding the subject they had to teach it as a series of meaningless tricks, the chief learning activity being repetition—*without* insight.

One hesitates to generalize on the findings of a study that involved only 476 persons, and 38,000 responses. Also one hesitates to generalize on the findings of a study, the data for which were gathered through the use of a group test. Far superior to this type of test perhaps would be the study of the behavior of each person individually through conversing with him,

and keeping anecdotal records of his performances on the test items. Realizing the hazards of generalizing on limited data, and keeping in mind the limitations of the study, we would nevertheless do well to consider the findings of the study as they impinge on the problem of the training of teachers of arithmetic.

One of the more significant findings of the study reported in this paper is that which is concerned with the degree of mastery of the basic mathematical understandings on the part of students in teacher-training institutions. This particular finding shows that, on the part of the students tested, there is no significant difference in achievement of basic mathematical understandings between the students who have taken a course in the Psychology and Teaching of Arithmetic and the students who have *not* taken such a course.

One might attempt to rationalize this finding by saying that the students already knew (understood) all of the arithmetic commonly taught in grades one through six, and therefore could not grow any more. An answer to this is in the finding which states that these students *understood* less than half of the arithmetic commonly taught in these grades.

One aspect of the needed redirection of the training of teachers of arithmetic seems to lie in the professional training offered in the teachers colleges and schools of education. This training, as it is usually set up at the present time, consists of a single course in the methodology of teaching arithmetic as a "tool" subject. Little emphasis is placed on the professional study of arithmetic as a science of numbers,—as a system of related ideas,—as a series of number relationships.

The findings of this study offer factual support for the recommendation made in the "Guidance Report of the Commission on Postwar plans."[6] In their report the commission states: "If you should wish to qualify for (a position in the teaching of arithmetic), the main requirement would be that you *understand* arithmetic. You cannot teach what you do not know."

"If you are planning to teach in the elementary school, three things are very important: (1) a good methods course; (2) at least one course that will give you subject-matter background for the mathematics taught in Grades I through VI; (3) at least one course that will give you subject-matter background for teaching the general mathematics of Grades VII and VIII."

Other significant findings of the study reported in this paper are concerned with the degree of mastery of the basic mathematical understandings on the part of teachers-in-service. These findings show that the teachers in the study *understood* about half (55%) of the arithmetic commonly taught in grades one through six, that they did not grow in understanding of the subject-matter of arithmetic by taking a graduate course in the Psychology and Teaching of Arithmetic; and that they did not grow in *understanding* of the subject matter of arithmetic as a result of experience in teaching arithmetic in the grades over a period of one or more years.

These findings seem to suggest several aspects of needed redirection in the program of in-service development of teachers of arithmetic. Curriculum revision of the professional courses must be concerned with emphasizing the subject matter *as well* as with the principles of teaching the subject-matter.

Very few persons in responsible educational positions today would doubt the worthwhileness of the educational values derivable from teaching arithmetic, or any other body of subject-matter, on the basis of meanings and understandings. In spite of the demonstrated values, however, understandings are little emphasized; this is particularly true in the field of arithmetic. Perhaps the reason for the

[6] "Guidance Report of the Commission on Postwar Plans," THE MATHEMATICS TEACHER, November, 1947, pp. 324–325.

general crass indifference to the teaching of meanings and understanding in arithmetic is due to the fact that arithmetic is still laboring under the handicap of being called a "tool" subject—a subject with no inherent understandings or meanings. Perhaps the reason for the general indifference is due to the application of a narrow interpretation (or mis-interpretation) of S-R psychology—an interpretation which stressed the need for immediacy of response, an interpretation which put a premium on accuracy, and an interpretation which made drill and repetition the chief learning activities. Whatever the reason—it is certain that the values of teaching for understanding have not been appreciated to the point of modifying the teaching and the curriculum to include the understandings, meanings, principles and generalizations basic to the number system.

Another possible reason for the comparative neglect in the teaching of understandings may be due to the fact that the persons responsible for the development of the curriculum materials did not themselves have a knowledge of the basic mathematical understandings. Findings of the study would tend to support this statement.

Since both carefully controlled research studies and other less-precise methods of rational inquiry have proven the worthwhileness of teaching for understandings, we must accept the responsibility for doing so. We are now in a position to create an "all-out" effort to discover new methods and materials for teaching and evaluating for growth in basic mathematical understandings. Dissemination of information about such methods and materials would do much to bring about on the part of the teacher and the administrator, acting in the role of curriculum makers, an increased awareness of the need for teaching and evaluating for growth in these understandings. Out of this increased awareness would come increased emphasis on understandings in textbooks, courses of study, teaching materials, and evaluation devices.

Understanding—This is the frontier of needed redirection in the training of teachers of arithmetic.

Letter to Santa Claus from a Math Teacher

Visit the people who struggle with Math,
Bring them courage, illumine their path.

Shower blessings on Polly Precision, please:
She never leaves out parentheses.

Increase my store of patience with 'em
When they're tangled in a logarithm.

Give me forbearance when (I quote)
"A hyperbola is an asymptote."

These are the faces I see each day,
May I kindle a light in each, I pray.

Katharine O'Brien
130 Hartley St,
Portland, Me.

Dramatizing Junior Mathematics*

By JULIA E. DIGGINS
Paul Junior High School, Washington, D. C.

CHILDREN of the 7th and 8th grades are very active. They are spontaneous—they lack self-consciousness—their humor is refreshing and they enjoy the "make believe."

Mathematics is dynamic and children should appreciate its activity and be stimulated by it. Dramatization can be the ground where the active child and the active subject meet.

Due to the fact that it is impossible to get a large group of children after school, a dramatization has to be done in class time and used as a teaching project. The material must come from the course of study.

Performed as a class project, the dramatization makes the whole group work closely together in a common undertaking. This usually carries over to other projects in the course of study.

Some children do not add well, but can speak well, or can draw well, or have a high degree of manual dexterity. If these talents can be brought to the aid of mathematics, that, too, carries over.

Many children who dislike mathematics develop a keen affection for the subject after they have given some lines successfully or drawn up a stage setting or made properties for a mathematics program.

I have found that the easiest programs for 8th grades have been imitations of Radio Quiz Shows. The assignment for home work is—Listen to Dr. I.Q. on the radio. (This is always a popular assignment.) Time it for questions, specialties and commercials and notice the manner of presentation.

All of our questions have to be on mathematics—most of them are serious—but it is always fun to have one or two like—"Why did Mr. Jones build his pig pen 5′ long, 10′ wide and 6′ high? Answer—"To put his pigs in," and "A butcher wears a size 16—32 shirt and is 6′ tall, what does he weigh?" Answer—"Meat."

The commercials are to sell mathematics using topics such as (1) the importance of orderly arrangement by number, (2) the practical uses of measurement, (3) mathematics in the arts, (4) mathematics as a universal and enduring language, (5) mathematics in nature, (6) mathematics in hobbies, and (7) the history of mathematics.

The next assignments are to write up questions and materials for commercials, and during class we read and discuss these questions and talks to determine whether they are similar to the radio questions and whether they are to the point and persuasive.

The next assignment is planning and timing our own program and being prepared to select the best efforts of the class in subject matter and performance. Every child is checked on each night's home work. It is important to get the cooperation of every child, because some very good ideas come from the laziest children.

The next day in class we vote for Master of Ceremonies, Announcer, Chairman of Committee on questions and the best commercials.

We called the program Dr. I. Quiz 'em. It took a week for preparation, used about fourteen members of the class—two on the stage, twelve ushers and brought in audience participation.

The same plan was used with "Information Please," which we called "Mathematics Please." This was an even better medium for teaching because we were able to bring in stories from the history of mathematics and to bring in musical, literary, political, sports and economic questions pertaining to mathematics. A school wide bulletin invited the whole

* Read at the twenty-seventh annual meeting of the National Council of Teachers of Mathematics at Baltimore, Maryland, April 1 and 2, 1949.

student body to submit questions and a bar of candy was the prize for a question muffed by the experts. The experts had to be very carefully picked from the best students.

For the 7A's, a review type performance is workable in emphasizing appreciation of mathematics, fundamental processes, simple geometric forms, per cents and measurements.

These topics were brought out by a play written by one class, called "Buddy's Bargain," based on a day without mathematics. The bargain was that Buddy would be excused from mathematics classes if he would agree to have nothing to do with anything mathematical. The bargain was entered into with great zest which dwindled by the hour, until at the end of the day he cancelled the bargain with even greater enthusiasm.

Children carrying large plane geometric figures and dressed in solid geometrical forms, danced and sang little tunes describing their characteristics. The girls prepared a fashion show of hats made of simple geometric forms and displayed them to the tune of the "Easter Parade." They were unusual and very stylish. Per cents and measurements were described in a series of illustrated talks.

This program took six weeks because teaching time and construction of figures had to go along with it. Seventy children took part.

"Fun with Fundamentals" was given as a musical by mixed 7th and 8th grade groups. The leading character was a boy who couldn't do his mathematics. A series of acts were given to help him. The decimal point and zero were the leading characters in an animated drill on short methods of multiplying and dividing by 10 and powers of 10 and changing per cents to decimals and decimals to per cents.

Lines of children with whole numbers and signs of operation marched in drills on the fundamental processes. Children with fractions and per cent equivalents marched together and separately to give practice in correct matching. Whenever it was possible we brought in original or borrowed songs which were appropriate. Illustrated talks were given on simple geometric forms and measurement of perimeters, areas, surface areas and volumes.

The story ended with the poor student very much accelerated. In fact he was able to do a very complicated problem in his head.

It is easier to get children to understand the meanings and to do the drill necessary for skill in mathematics after they have gotten a better perspective of the great scope of the subject.

As one practices his exercises and scales to play beautiful melodies, so the child should practice his fundamentals to gain technique in a subject that—Turns the wheels of industry, builds bridges, designs airplanes, dominates transportation and rings the cash register.

To Teachers of Elementary, High School, and College Mathematics

The Committee on Instructional Aids needs your assistance in preparing an exhibit and demonstration of instructional aids for the twenty-eighth annual meeting of the National Council of Teachers of Mathematics to be held in Chicago, April 13–15, 1950.

We know that many of you and your students have constructed instruments and other types of multi-sensory aids for illustrating mathematical principles. If they were on display during the convention and could be seen and manipulated we are certain that everyone visiting the exhibits would benefit thereby.

We would be pleased, therefore, to hear from anyone who would like to display such materials. Any communications should be addressed to

Mr. Reino M. Takala
Hinsdale Township High School
Hinsdale, Illinois

Have You Heard?

By Edith Woolsey
Sanford Junior High School, Minneapolis, Minn.

Are you looking for some more topics for use at your local mathematics club meetings? Here are a few more summaries of discussions that were held at the Baltimore meeting in April. Do the teachers in your club have different ideas about these subjects?

I. Workbooks, Their Use and Abuse.

Leader: Mrs. Marie S. Wilcox, Washington High School, Indianapolis

Four ways of using a workbook were stated:

1. As a main text book
2. Drill
3. Review
4. Supplementary material

An abuse of the workbook would be to let the children do all the work, and the teachers do little. Workbooks should not be used as text books but they are useful for supplementary or drill material.

There are some advantages in using a workbook:

1. One has material at hand.
2. It is an aid if used to start work on entering the room.
3. It makes it possible to give individual assignments to children when the class is very large.
4. The money investment is small.

II. How Can We Provide Better Coordination between Our High School and College Programs?

Leader: Florence E. Loose, Wilmington High School, Wilmington, Delaware

After some discussion, the group made the following recommendations:

1. The Guidance Pamphlet for High School Students should be in the hands of each high school student and each college freshman. A guidance pamphlet should be developed to meet the needs of each grade group, from junior high school to college juniors. The group also recommends guidance pamphlets for parents.

2. Frequent conferences should be held with each individual, stressing the importance of mathematics in all fields of education and the opportunity that each student has in his chosen field.

3. We should adopt a more sympathetic understanding of college students during their period of adjustment. Most new college students need organized help of some type in methods of study, correct reading habits, note-taking procedures, rules for thinking, memory devices, and an appreciation of the role that mathematics plays in our scientific age. They also have additional personal problems to be solved and new social adjustments to be made.

4. A good mathematics vocabulary should be developed beginning in the elementary grades.

5. High Schools chould acquaint students with longer college examinations. A spring tournament, with teams from several schools, was suggested as one method.

6. The group recommends a three track program for mathematics, similar to that which is now being followed at Wilmington, Delaware. This program requires four years of mathematics for every student. The work is adapted to the needs and ability of the student.

7. If possible, each student should be made to realize that he is responsible for his success, that the initiative is his.

III. Materials and Devices for an Experimental Approach to Statistics.

Leader: Paul C. Clifford, State Teachers College, Montclair, N. J.

In discussing an experimental approach to statistics, Mr. Clifford drew on his experience in teaching statistics in high school, in teaching seven-day courses in quality control in industry, and on his

experience as a statistical consultant in industry. He described experimental techniques used in industrial courses to introduce men entirely unfamiliar with statistics to basic statistical concepts and their use in quality control.

The question as to whether understanding of concepts depends on skills in related computations was discussed. There seemed to be agreement that understanding of concepts could be had without computational skills, and that the concepts, with only a minimum amount of simple computation, could be useful in practice.

The group felt that it was very important in any statistics course to have problems and data that were interesting and meaningful to the class.

IV. General Mathematics in College

Leader: Two groups met to discuss this topic. Their opinions have been summarized in the statements following. The leaders were: Charles H. Butler, Western Michigan College of Education Kalamazoo, Mich., Phillip S. Jones, Western Michigan College of Education Kalamazoo, Mich.

If general mathematics courses are to have a fair chance to establish their validity and worth, there needs to come about some widespread acceptance not only of the place of such courses in the educational program, but also of the objectives and the topical content of these courses. There needs to be a definitive statement of the aim and the content of any such course and this statement needs to have relatively wide acceptance if the course is to have a fair chance to succeed.

More adequate textbooks are badly needed for such courses.

Serious efforts are needed to bring about widespread agreement on definitive statements of aim and of topical content. Such agreement would pave the way for more satisfactory text books and would cut down the present confusion about what should be included in the course.

Both discussion groups passed, as a matter of record, a motion recommending that the National Council appoint a committee to make an extensive and significant study of the aims and content of courses in general mathematics at the freshman-sophomore level in college, having in mind students with different degrees of mathematical background. This committee should make a report at the annual meeting in 1951.

National Teacher Examinations Will Be Held on February 18, 1950

PRINCETON, N. J., October 22. The National Teacher Examinations, prepared and administered annually by Educational Testing Service, under sponsorship of the American Council on Education, will be given at testing centers throughout the United States on Saturday, February 18, 1950.

At the one-day testing session a candidate may take the Common Examinations, which include tests in General Culture, Mental Abilities and Basic Skills, and Professional Information; and one of eleven Optional Examinations, designed to demonstrate mastery of subject matter to be taught. The college which a candidate is attending or the school system in which he is seeking employment will advise him whether he must offer the National Teacher Examinations and which of the tests he should take.

Application forms, and a Bulletin of Information describing registration procedure and containing sample test questions, may be obtained from college officials, school superintendents, or directly from Educational Testing Service, P. O. Box 592, Princeton, New Jersey. A completed application, accompanied by the proper examination fee, should reach the ETS office not later than January 20, 1950.

Miscellanea—Mathematical, Historical, Pedagogical

By PHILLIP S. JONES
University of Michigan, Ann Arbor, Michigan

LITERARY MATHEMATICS

ANYONE interested in mathematics enjoys meeting mathematical allusions in non-mathematical settings even when irked by their inaccuracy or lack of true aptness. Such allusions may often be used to enliven a class, to illustrate a point, or to motivate a more serious discussion, as well as to indicate that the concepts and vocabulary of mathematics are a part of our culture and appropriate material for the general education of all intelligent citizens.

When Eliot Janeway in an article on John Bricker, then a vice presidential candidate, referred to "the fetchingly non-Euclidean universe of Ohio politics" on page 106 of *Life* magazine for November 6, 1942, he was probably trying to convey an impression of illogicality and unpredictability that is quite antithetical to the nature but not to the popular conception of non-Euclidean geometry.

Samuel Grafton in his syndicated column *I'd Rather Be Right* for November 17, 1948, showed a better concept of non-Euclidean geometry and carried along an interesting analogy when he wrote:

> "Yesterday I speculated for about seven hundred words on what would happen if we were to change the basis of our foreign policy, from the idea that we must hastily prepare against the danger of war with Russia, to the entirely different idea that Russia can't make war upon us, even if she should want to, because she is too battered, too poor and too weak.
>
> "(I admit it is a novel idea that there is no real prospect of war. Yet, as I pointed out yesterday, certain modern mathematicians have done rather wonderful things by chucking out such well-established ideas as Euclid's notion that parallel lines can never meet. They have thus built a non-Euclidean geometry which has, in its own way, been useful to the world. What I propose, speculatively, is a kind of non-Euclidean foreign policy, built on some other basis than the commonly accepted one.)
>
> " . . . If we would change the basic assumption of our foreign policy to the novel idea that there is going to be peace . . . "

Although one may take exception both mathematically and otherwise to the comments of George W. Martin under the heading "Another Man's Poison" in *Harpers* for September, 1944 (page 357 ff.), nevertheless one may find his use of mathematical terminology and history of some interest. He says:

> "It is not for nothing that an all-wise Providence has created all men equal and women irrational. In mathematics it is the irrational numbers which cause the conversation. Who cares about the integers which follow one another with such monotonous reliability? Who cares whether Achilles will catch the tortoise? Irrationals are not bound by facts or logic. Every woman knows to a certainty that the tortoise will be defeated. It is of no avail to argue about the matter. What is a hypothesis in the face of experience? There may be no such thing as the square root of minus one. What of it? It is a useful concept in connection with certain operations; and if it is useful, let us get on with our knitting."

Not even the comics nor sports are free of mathematical literary allusions for we find that in the comic strip *Barnaby*, Atlas, the Mental Giant, remembers Mr. O'Malley's name by the formula

$$J_1(0)\,(e^{\pi i}+1)+M\int_0^A dx+\lim_{N\to 10}\begin{vmatrix}\log N & & y\\ & {}^{-1} & \\ -\ \mathrm{lu}\ 1 & & \end{vmatrix}$$

and carefully computes on his slide rule the square root of 49 to be 6.998.

A Purdue University yell according to *The Detroit News* of December 21, 1946, is "E to X, DY, DX—E to the X, DX—Cosine, Secant, Tangent, Sine—Three Point One Four One Nine—Square Root, Cube Root, BTU—Slipstick, Slide Rule, Yea Purdue."

Won't you readers send me some of *your* pet mathematical-literary allusions?

Applications and the Parallelogram

There are several good discussions of the uses of properties of parallelograms in every day life,* but new variations on this old theme appear regularly. At the Baltimore meeting of the National Council there was exhibited the ingenious combination of gears and a variable parallelogram by means of which the rotary motion of the handle on your car door is transformed into the translatory motion of the window sliding up and down. (Won't someone send me the name of the exhibitor so that he may be given credit?) When the writer asked to examine this mechanism at the stock room of his local car dealer the stock clerk found an old slightly defective assembly and gave it to the mathematics department.

A new tail gate for trucks is in use in this area. This Daybrook Hydraulic Power Gate can be rotated from its vertical position down to a horizontal position level with the truck bed and then it can be made to lower and raise itself to and from the road level in this horizontal position. In other words the tail gate is also a portable loading and unloading elevator. Its horizontal position is maintained by fastening the tail gate at right angles to one side of a parallelogram the opposite side of which is held in a vertical position beneath the truck. The rotating of the tail gate from the horizontal to the vertical position is accomplished by rotating this same opposite side. The manufacturers, The Daybrook Hydraulic Corporation of Bowling Green, Ohio, will send an advertising folder and installation drawing on request.

The least that the display of such applications can do is to stimulate the interest of your geometry class. Other pedagogical uses for applications and models will be suggested next month.

* R. C. Yates, "The Story of the Parallelogram," The Mathematics Teacher, XXXIII (April, 1940) p. 179 ff.

———, "Linkages," *The Eighteenth Yearbook of the National Council of Teachers of Mathematics*, pp. 127–9.

National Council of Teachers of Mathematics, *The Seventeenth Yearbook*, pp. 192–3.

P. S. Jones, "Multi-Sensory Aids Based on Applications of Mathematics," The Mathematics Teacher, XL (Oct. 1947) pp. 285–6.

Group Affiliation With the National Council

By H. W. Charlesworth,
Chairman of Affiliated Groups, East High School, Denver, Colorado

This is the third in this series of articles. I shall assume that you have read the articles on affiliation in the October and November issues. As the Chairman of Affiliated Groups, appointed by the Board of Directors on March 30 of this year, I have been requested to present recommendations on reorganization of affiliation to the Board at the Chicago meeting next April. It is our desire that these recommendations come from the groups themselves. We hope to accomplish this through the means of the questionnaire that I sent out to affiliated groups in September and through the Delegate Assembly. Remember, if affiliation of your group is completed by March 1, 1950, you are entitled to send *one* delegate to the Delegate Assembly in Chicago next April.

Recent Affiliations and New Groups:—The situation as of September 10, the time of writing this article, is as follows. Two new affiliations have been completed this year, The Mathematics and Physics Section of the Ontario Educational Association of Ontario, Canada, and The Hillsborough County Mathematics Association of Florida. Many state organizations are under way. In 1948 the first steps were taken to organize the Mathematics Department of Florida Education Association into a Florida Council of Teachers of Mathematics; reorganization was completed in March of this year with a membership of about three hundred. This group plans to affiliate with the National Council. The Illinois Council of Teachers of Mathematics has been organized and will have its first annual meeting in October of this year. They plan to affiliate with the National Council. Wisconsin is organizing a state council of teachers of mathematics with the purpose of affiliating. A state council was started in September of last year in Minnesota, their first meeting being held last April. They are interested in affiliation and will take action on the matter in September. Indiana is now organizing a state council of teachers of mathematics which will affiliate. The Arizona Association of Teachers of Mathematics recently organized and will affiliate soon. West Virginia is considering the organization of a state group in November. Pennsylvania hopes to complete the organization of a state council at Christmas time of this year at Harrisburg. Utah will take up the matter of a state council this fall, also Colorado and North Carolina. Organization of southern Oregon is being considered and Virginia will consider in October the affiliation of their state association of mathematics teachers. Northeastern Ohio is planning to organize and affiliate. The Mathematics Teachers Association of New York City was to consider affiliation in May of this year. No doubt, there are others, but these I have learned about through correspondence and at recent conventions of the National Council. And, of course, there will be still others by the time this article appears Please keep me informed.

How to Affiliate With the National Council:—*First,* organize your group, frame a constitution and by-laws, elect officers, select committees, etc. *Second,* have the group pass a resolution expressing their desire to affiliate with the National Council. *Third,* fill out the application form which you may secure from me. This application will request such information as: name of organization, brief history of organization, number of regular meetings per year and average attendance, time and place of meetings, territory served, number of members and number who are also members of NCTM, amount of local dues, names and addresses of officers. The application form will ask such

questions as these: Do you have a written constitution? Do you have a list of members with teaching positions and addresses? Do you have a board of directors or the like? Do you have a coordinating committee? Does your group publish an official journal or bulletin? Do you have a plan for continuance of leadership? *Fourth*, if affiliation is completed by March 1, 1950, and you have received your charter, elect a delegate to represent your group at the Delegate Assembly in Chicago. At present there is no fee for the issuance of charter and no affiliation dues. These matters are among those which will be considered by the Delegate Assembly next April.

Where to Get Help:—State representatives have been asked to present the matter of organizing a state council of teachers of mathematics in their respective states where such groups do not already exist. They will give any assistance they can. There is much help to be gained by requesting it from organizations that have been established for some time as well as from those groups that have recently gone through the process of organizing. It may be of help to have someone representing the National Council meet with your group to present and discuss the matter, one of the officers or a member of the Board, your state representative or the chairman of state representatives, or myself. You might obtain the services of one of these who is close to you. Write directly to that person or write to me about it. It is important to get the right start. The right leadership is quite important from the very beginning. A poorly planned organization or one headed by weak leadership will not find a cure for all such troubles merely by affiliating with the National Council. It may take longer than you would like to get ready for affiliation, but it might be better to progress slowly.

Some of the Oldest Affiliated Groups:—It will be of interest to note some of the earliest groups to affiliate with the National Council. We shall list the first eight, those affiliated in 1929, the first year of affiliation activities. The Men's Mathematics Club of Chicago and Metropolitan Area holds the honor of being No. 1, having received its charter on June 6, 1929. The next seven are: No. 2, Alpha Mu Omega, Peru, Nebraska; No. 3, Louisiana-Mississippi Section of the National Council of Teachers of Mathematics; No. 4, Detroit Mathematics Club; No. 5, Columbus Mathematics Club; No. 6, Section 19, Secondary Mathematics of the New York Society for Experimental Study of Education; No. 7, Women's Mathematics Club, Chicago and Vicinity; No. 8, Colorado Chapter of the National Council of Teachers of Mathematics. Incidentally, the list of affiliated groups given in the October 1948 issue of THE MATHEMATICS TEACHER is not complete and is otherwise inaccurate. We plan to publish a list of affiliated groups in the March 1950 issue. As has been stated in the two preceding articles, this list is to include all groups that have renewed affiliation by December 1, 1949, and also new groups which have completed affiliation by that date.

Address all inquiries about affiliation to me. Comments and suggestions are welcome at any time.

CORRECTION

The Eberhard Faber Pencil Company, 37 Greenpoint Avenue, Brooklyn 22, N. Y., has notified us that three of the charts described in the October 1949 issue, p. 315 are no longer available: "Basic Principles in Industrial Design," "Basic Principles of Triangulation," and "Standard Woodworking Joints." Another chart is available, "Standard Symbols for Wiring Plans."

In the same issue, the address for B.25 was incomplete; it should read "Mason City, Iowa."

New Jersey Representative—The National Council of Teachers of Mathematics*—Mary C. Rogers, Westfield, New Jersey

At the early age of five, Mary C. Rogers expressed an intention to teach, from which she has never deviated. Her early professional training was received at the State Teachers College, Mansfield, Pennsylvania, where she graduated magna cum laude. She has done graduate work at Cornell University and the School of Education, New York University.

Miss Rogers' entire teaching experience in New Jersey has been at Westfield, where she is Chairman of Mathematics in the Roosevelt Junior High School.

Miss Rogers' professional interests and activities have been rather numerous and varied. She served on a New Jersey Syllabus Committee in setting up a program of mathematics for the Ninth Grade General pupil. At present she is a member of a State Curriculum Committee under the direction of Dr. Heber H. Ryan, Assistant Commissioner of Education for Secondary Schools. This committee is evaluating existing programs of study in Secondary School Mathematics and making recommendations as to needed curriculum revisions and additions.

In 1947 Miss Rogers represented the National Council of Teachers of Mathematics on a special committee working with Commissioner Studebaker and some of his assistants in the United States Office of Education. This committee made recommendations regarding printed instructional materials considered essential for optimum instruction in the various fields of Secondary School Education. Miss Rogers taught at the 1947 Mathematics Institute at Duke University, conducting a Study Group in Junior High School Mathematics and assisting with a group in Tests and Measurements.

Mary C. Rogers

Miss Rogers has been very active in affairs of the Association of Mathematics Teachers of New Jersey, having served as both Vice-president and President. During her administration the New Jersey Association were hosts to the National Council Convention at Atlantic City. Miss Rogers coordinator of all committee activities. She has been Editor of the State Association publication and is now Secretary-Treasurer. Beside working on numerous temporary committees, Miss Rogers has served on the following standing committees:

1939–40. Chairman of the Committee on Revision of the State Association Constitution.

* From time to time, The Mathematics Teacher will publish articles on the different State Representatives. This is the first of this series. W.D.R.

1944–47. Chairman of Junior High School Research, under the direction of the Post-War Policies Commission of New Jersey. This Commission was set up by the Association of Mathematics Teachers of New Jersey with Dr. Howard F. Fehr as general coordinator. This State Commission worked with the Post-War Policies Commission of the National Council with Dr. Virgil S. Mallory as liaison officer between the Council and the New Jersey Association.

1942–50. Membership Chairman. During this time membership has been more than doubled. The present total is 870. Much of this growth is the result of the popularity of "100% Schools" and "All But One Schools" stimulated throughout the schools of the state. Teachers of mathematics are very proud of being members of these Honor Schools. Over 100 schools report 100% membership each year. Twenty or thirty additional schools report all teachers of mathematics but one as Association members.

Miss Rogers has published articles in THE MATHEMATICS TEACHER, *The New Jersey Mathematics Teacher*, *The National Safety Magazine* and *Rho Journal—Pi Lambda Theta*. She has appeared on the programs of numerous State Association meetings. She has lectured or led discussion groups at a goodly number of National Council Conventions. She spoke at a meeting of Section 11—Metropolitan Group for the Experimental Study of Mathematics Education. She presented The Role of the Junior High School in Mathematics Education at a meeting of the Metropolitan Section of the Mathematics Association of America.

Miss Rogers has been a member of the National Council for nineteen years. Since 1942 she has been New Jersey Representative. Special National Council committees on which she has worked, or is now working, include: Chairman of Hospitality at the Atlantic City Convention, 1947; Committee on Affiliated Groups; Committee on Contests and Scholarships; Committee on "100% Schools" and "All But One Schools"; Committee on Programs of Study in Junior High School Mathematics.

AIDS TO TEACHING

By

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BOOKLETS

B.27—A Career as Actuary

American Institute of Actuaries; 135 South LaSalle Street; Chicago, Illinois

Booklet: 8 pages; 1946; free.

Description: This pamphlet describes concisely the character of actuarial work and the opportunities for employment and advancement in this field. Qualification as an actuary is attained in the process of passing a number of examinations required for membership in a recognized actuarial body. Since the qualities required for success are described as "competent statistical and mathematical capacity, adequate economic and financial knowledge, and wide social information," college courses are suggested that will develop these qualities.

Appraisal: Although this pamphlet is designed for college students, it will be valuable material for the mathematics teacher to use in counseling her students with high aptitude and interest in mathematics. It is written in a simple direct style that will appeal to high school students. The pamphlet emphasizes the opportunities for employment but it does not give specific figures as to salaries or number of positions. Although it outlines the training and examination required, it does little to point out how rigorous the requirements are.

CHARTS

C.11—Wall Charts on Map Projections

Denoyer-Geppert Company; 5235–37 Ravenswood Avenue; Chicago 40, Illinois

44×58″, $16.25.

Description: The set consists of three red, black, and white charts which illustrate common projections of the globe upon a plane. The charts are attractively printed on heavy manila paper. Each shows not only the resulting maps but also the more pertinent methods of projection employed. The contents of each chart are discussed individually below:

"Hemisphere Projections," (J-208). This chart deals with geometric projections. The basic types included are the Orthographic, Stereographic, Globular, and Gnomonic; secondary projections illustrated are the Polar Equidistant, Polar Equal Area, and Lambert Equal Area.

"World Projections," (J-209). Illustrations are provided for each of the following: Gores of a Globe, Gnomonic Cylindrical Projection, Gall's Stereographic, the Homolographic, the Sinusoidal, Goode's Homolosine, and Denoyer's Semi-elliptical projection.

"Conic Projections," (J-210). The following projections are developed: Simple Conic tangent at one point, Conic tangent at two standard parallels, Lambert's conformal, the Polyconic, Albers', and Bonne's.

Appraisal: The basic mathematics of cartography is within the understanding of secondary students. The geometric projections are not discussed incidentally in these charts; they are an excellent device for teaching the concepts involved and are not as bulky or expensive as the models used by the Navy. If purchased, they should be displayed singly at first in order to minimize the danger of presenting too much information at one time. The charts

are extremely expensive and purchase of them is probably not justified unless they are to be used by the Social Studies department also. (Reviewed by Bernard Singer, Hyannis, Massachusetts.)

EQUIPMENT

E.23—Calculation

Miss Claire Walker; 728 De Witt Avenue; Sanger, California

Card Game; 52 cards, $2\frac{1}{4}\times3\frac{1}{2}''$, plus one direction card; in box; $1.10.

Description: There are 13 sets of cards (numbered from 1 to 13) with four in each set. Each card has the same heading and contains four questions; each of the four questions is answered on one of the cards of the book, and appears at the top of the list; the other three questions are unanswered. The game is played similar to "Authors" by trying to complete books. When you request a card from another player you must ask for it by completing one of the uncompleted questions on a card of the book you hold.

Appraisal: There are, and always will be, places in mathematics where drill on facts is necessary. For that particular part of mathematics teaching, games such as this are excellent as long as they do not give the appearance to either teacher or pupils that they serve the only end of learning mathematics. The title of the game is not well chosen for many of the facts are based on geometric definitions, facts from commercial arithmetic and simple formulas. The cards are simple, but sturdy and attractive.

FILMS

F.41—Principles of Scale Drawing

Coronet Instructional Films; Coronet Building; Chicago 1, Illinois; Educational Collaborator: Harold P. Gawcett.

16 mm film; Black and white or color; Sound; 1 reel; $45 (BW), $90 (color); 1948.

Content: It is the purpose of this film to give instruction in the skills and interpretation of scale drawing by showing how to determine scale, how to use measuring and scaling tools, the meaning of terms and the importance of scale drawing in modern industry. The film deals with the problems of three students who are making a booth for a school carnival. The film opens by showing sketches of a booth for a carnival which the students of a certain high school are planning. Due to the lack of a plan and an inadequate sketch, it is necessary for them to make a complete drawing of their booth. The film then points out that a scale drawing should indicate the materials as well as being a proportionate drawing. In making the drawing, close-ups are shown of the use of various drawing instruments. This plan is then used to cut materials and build the booth. The application of scale drawing in mass production and in industry is also illustrated.

Appraisal: This film uses a realistic high school situation to develop principles of scale drawing and to show the benefits and uses of scale drawing in an actual problem. Although the close-up photographs of the use of drawing instruments are quite clear, there are distracting shadows. It is somewhat inconsistent to use a high quality divider with an ordinary 12 inch ruler or to make three exact scale drawings to construct a simple booth. However, the treatment of the topic is well unified and probably as complete as possible in one reel. The photography and commentary are very satisfactory except that dialogue by the students rather than discussion by a commentator would have been more realistic. Mathematics teachers should find this film very useful in introducing students to scale drawing.

F.42—Algebra in Everyday Life

Coronet Instructional Films; Coronet Building; Chicago 1, Illinois.

Educational Collaborator: R. Orin Cornett.

16 mm film; Black and white or color; Sound; 1 reel; $45 (BW), $90 (color); 1948.

Content: This film shows how two students solve a problem in paint mixing by the use of algebra. In painting three out of five panels, 32 ounces of red paint and 16 ounces of yellow paint were used. The problem of how much paint to mix for the remaining two panels is solved by writing two formulas using two unknowns. The steps required in the solution of this problem, namely, (1) observation, (2) translation, (3) manipulation and computation, are then applied to a simple problem in determining the number of pieces of candy originally in a candy box. The film also shows how algebra is used in everyday life as well as in specialized fields such as railroading, electricity and food nutrition. This is followed by a re-enumeration of the three steps involved in attacking an algebraic problem.

Appraisal: Although the purpose of this film is to show the everyday uses of algebra, it is doubtful that it will satisfy ninth-graders asking "Why do we study algebra"? The use of algebra instead of arithmetic to solve an extremely simple problem on the number of pieces of candy in a box will not appeal to pupils. It would seem that an appropriate time for the showing of a motivational film such as this would be at the beginning of ninth grade algebra. However, the use of two equations in two unknowns to solve a problem is not appropriate so early in the course. It is doubtful that a paint mixing problem as illustrated would ever be solved in this manner by a painter. It is the opinion of the reviewer that more realistic applications of algebra could have been used, for example, the flying of an airplane, the throwing of a basketball, or the amount of energy in an atomic bomb. The photography and commentary are very satisfactory with the use of paint on a panel instead of chalk on a blackboard furnishing an interesting variation for presenting written material.

FILMSTRIPS

FS.58—Lines

Jam Handy Organization; 2900 East Grand Boulevard; Detroit 11, Michigan

35 mm filmstrip; color; 27 frames; $4.95.

Content: Simple but colorful drawings are used in this filmstrip to show how different lines can be used to express moods such as anger, gracefulness, or playfulness. Several illustrations are given of lines that express an idea by themselves. The filmstrip also shows how to make lines used in designs seem short, busy or important.

FS.59—Shapes

Jam Handy Organization; 2900 East Grand Boulevard; Detroit 11, Michigan.

35 mm filmstrips; color; 29 frames; $4.95.

Content: This filmstrip shows how shapes are made by lines and how different lines can be used to make shapes that are graceful, fat, skinny, scarey, proud, or wiggly. By changing the shape of a square or a circle, a variety of shapes are produced. Finally, many different shapes are combined to form new, unusual or attractive shapes.

FS.60—More Shapes

Jam Handy Organization; 2900 East Grand Boulevard; Detroit 11, Michigan.

35 mm filmstrip; color; 24 frames; $4.95.

Content: This filmstrip begins by showing how squares can be combined to give a three-dimensional effect. Squares are then combined with straight or curved lines to give new effects. The size and position of squares are changed to form many interesting designs. A variety of unusual patterns, each giving a different impression, are formed by combining different shapes and colors.

FS.61—Solid Shapes

Jam Handy Organization; 2900 East Grand Boulevard; Detroit 11, Michigan.

35 mm filmstrip; color; 24 frames; $4.95.

Content: By using pictures with appropriate associations, this filmstrip shows three-dimensional representation with cubes, spheres, and prisms. For example, a ribbon with a bow tied around a sphere shows very clearly the three dimensional effect of a sphere. The effect of shading to give depth is also shown. The relation of feeling to shapes is illustrated by such items as circles, pyramids, and twisted objects.

Appraisals of FS.58, FS.59, FS.60, and FS.61: Although these filmstrips are designed to illustrate and explain in simple terms some of the important elements of art for grades 4 to 9, they contain principles and designs that will be related to work in the mathematics class. The titles "More Shapes" and "Solid Shapes" would be appropriate in the solid geometry class where students have difficulty "seeing" three-dimensions. The entire series will be useful in showing the relation of geometric figures to art. However, it should be kept in mind that the series is essentially a series for teaching elements of art and thus, will be primarily for enrichment in the mathematics class. The drawings and designs included are simple yet attractive.

INSTRUMENTS

I.15—Yoder-Lafayette Sextant (No. 10)

Yoder Instruments; East Palestine, Ohio. Arc; 7″ radius; $22.50.

Description: The die-cast frame is connected to an etched aluminum arc whose clear black graduations are calibrated to half degrees. The index arm contains an engraved vernier which increases the precision to the nearest three minutes. An adjustable screw is used to clamp the index arm to the fibre runner when a reading has been taken. Both the index and horizontal mirrors are adjustable and copper-backed. The telescopic sight tube is likewise adjustable; removable sun screens are provided for decreasing glare while "shooting" the sun. The mahogany handle and brass screws used in assembling the sextant are corrosion-resistant. Superior physical qualities indicate that pains have been taken to produce a precision instrument.

Appraisal: The instrument may be used advantageously by teachers who desire a fairly reliable sextant. It may be used to determine latitude and longitude, to locate stars and planets, and to estimate the distance between inaccessible points.

The instrument is essential to the navigator but does not require a great deal of mathematical ability. This particular sextant is superior to those which can be made by students; it must be handled with care to insure correct alignment of the mirrors and telescope and to prevent shattering the mirrors. A wooden case is provided for protecting the sextant while not being used.

The cost of the instrument is somewhat high, due to recent increases in price, but may not be excessive when a fairly precise sextant is desired. (Reviewed by Bernard Singer; Hyannis, Massachusetts.)

MODELS

M.8—Busts of Newton and LaPlace; Three Statues of Mathematicians

Caproni Galleries, Inc.; 1914–20 Washington Street; Boston, 16, Massachusetts

Description: The Caproni Galleries produce three busts which are of use in the teaching of mathematics. They are molded of plaster of paris and each has a sturdy base. The busts are quite life-like. The likeness of Newton is available in two sizes. Model number 5438 is 2′6″ tall; its price is $37.50. The smaller size, number 5760, is 1′10″ tall and costs $15.75. Both models are exactly alike with the exception of size; they show the head and shoulders of Newton.

The bust of LaPlace, Number 5479, is available in only one size—2′4″. Its price is $30.00.

Appraisal: Statues are not frequently found within schoolrooms, small busts may, however, add to the teaching atmos-

phere fully as much as pictures, posters and charts. The larger busts discussed above are quite expensive and demanding of space but the smaller model of Newton has neither of these objections.

The work of LaPlace in the fields of analysis and celestial mechanics is quite advanced for secondary students and the bust of this mathematician is probably not appropriate in the high school classroom. Newton's achievements with respect to the binomial theorem and the beginnings of differential and integral calculus may be considered within the scope of many secondary curricula; consequently, his bust may be used appropriately.

The busts may be placed upon a table or strong shelf in the classroom or a pedestal may be purchased from the Caproni Galleries for approximately $20.00. The latter suggestion is hardly necessary if the smaller model is to be purchased. (Reviewed by Bernard Singer, Hyannis, Massachusetts.)

PICTURES

P.7—The University Prints (C 107, 168, D 153, 287, 405)

University Prints; 11 Boyd Street; Newton 58, Massachusetts

$5\frac{1}{2}\times 8''$; $.02 each (minimum order is $.25.)

Description: This set consists of five inexpensive reproductions of mathematical pictures. They are printed in black and white with captions which signify the title artist, and school to which he belonged.

"School of Athens" (C 167). This painting by Raphael depicts mathematical activity in an ancient Greek gathering place. The scene is one of varied activity; Plato and Aristotle are shown conversing, both men surrounded by disciples. Pythagorus is at work in another section of the picture and Socrates is placed between Plato and Pythagorus to represent the connecting link between their two schools. Archimedes explains a geometric problem to several students as Ptolemy and Zoroaster, each holding a sphere, converse. More than fifty people appear in the picture and each has special significance[1]—mathematical, philosophical, or merely to emphasize the importance of the preceding groups.

"Mathematicians" (C 168). This is a detail reproduction of a section of the aforementioned painting. It is a closeup of Archimedes explaining the mathematical problem to the students as Ptolemy and Zoroaster are viewed by Perugine and Raphael (who have no mathematical significance.)

"Portrait of Galileo" (D 153). Sustermon's painting is an extremely attractive and significant likeness of Galileo but loses its effectiveness because it is a posed picture and does not represent an activity which would distinguish Galileo as an astronomer, physicist, or mathematician.

"Portrait of a Mathematician" (D 287). This painting portrays a dignified mathematician pointing to a geometric problem which he has drawn on a blackboard. "It seems surprising that the name of a Geometrician, who undoubtedly was of some celebrity in the time of Ferdinand Hol, has not been handed down for posterity with his portrait.[2] This picture is less effective because of its anonymous subject.

"Melancholy" (D 405). Albrecht Dürer's engraving depicts geometry as a melancholic winged lady who ponders over a mathematical problem. This sad mood was considered appropriate for the artist felt that mathematicians, being more cognizant of the insignificance of man, were quite prone to experience long periods of dejection. The lady is surrounded by a number of objects to intensify the geometric atmosphere. Among these are a compass, polyhedron, and sphere which symbolize pure mathematics; a magic square, hourglass, drawing instruments,

[1] For an excellent analysis of the personalities in this see: Charles C. Perkins, *Raphael and Michelangelo* (Boston: James R. Osgood and Co., 1878) pp. 124–128.

[2] Duchesne Aine, Recueil des Plus Beaux Tableaux, Statues, et Bas-Reliefs qui Existaient au Louvre avant 1815. (Paris: et W. Galignani); p. 66.

bell, and pair of scales represent applied geometry.

Appraisal: These pictures may be used advantageously to contribute toward the creation of a mathematical atmosphere in the classroom. They also present a direct approach to the study of the mathematics of art since not only the forms and arrangements within the pictures, but also the contents are significant.

The pictures are adaptable to group study either by using an opaque projector or by purchasing the lantern slides ($3\frac{1}{4}'' \times 4''$ or $2'' \times 2''$) which may be had on these subjects. The slides cost $.68 each plus postage. There can be little doubt that the pictures are well worth the price of $.02 each. (Reviewed by Bernard Singer, Hyannis, Massachusetts.)

SOURCES OF MATERIAL FOR LABORATORY WORK

SL.10—Levers

Educational Radio Script Exchange; United States Office of Education; Federal Security Agency; Washington, D. C.

Radio Script; 8 pages, mimeographed (15 minutes); 1945, Free on loan.

Description: This program was originally part of the "Science Story Teller" Series produced by the Chicago Public School of the Air. It is not really a dramatization, but a radio story told by one person who asks his audience to participate by balancing rulers, books, etc. and then developing the vocabulary: lever, fulcrum, resistance, effort, class of lever and lever arm. Then the three classes of levers are discussed and illustrated, and the other simple machines besides levers are mentioned by name.

Appraisal: This is a very good example of one of the worst types of radio lesson. The attempt to accompany explanation with participation is very necessary to learning, but should be directed by a teacher on the spot; radio lessons have other functions, such as historical background material, which they can perform very well. Rather than this "master teacher role" over the radio, the material can serve as a model inductive lesson for each teacher to study. Dramatizations, uses of more than one voice, could be written and performed by the pupils themselves to cover the same material and to give some of the times in history when levers have been important.

SL.11—The World is Yours—The Romance of Clocks

Educational Radio Script Exchange; United States Office of Education; Federal Security Agency; Washington, D. C.

Radio Script; 27 pages, mimeographed (30 minutes); Free on loan.

Description: This story was produced by the office of Education in cooperation with the Smithsonian Institution and the National Broadcasting Company. Although 27 characters are mentioned in the cast, many may be doubled in production. Two children visit the Smithsonian and hear the story of clocks: the sundial, water clock, candle clock, DeVick escapement clock, pendulum clock and the chronometer. This is told by dramatizing incidents about primitive men, Alfred the Great, Galileo, Professor Hooke, Sir Isaac Newton, John Harrison and Benjamin Raymond. One page is devoted to a list of the cast and description of their voices, and three pages to sound effects and other production notes.

Appraisal: This script is an excellent, usable, practical example of educational broadcasting. The discussions of accuracy, error of measurement and instruments for time measure should show any mathematics class the applications of the terms used, and should be of interest from about the fifth grade through the twelfth. The production of this would be more cumbersome than a simpler play, but, once produced, it could be recorded for future use and another produced each year. Facts could be checked, posters made to advertise the play, models of the simpler time-devices made, and other correlated activity if the curriculum can allow it.